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AN ANALYSIS OF THE RESULTS OF FACTORIAL EXPERIMENTS IN MINERAL PROCESSING WITH COMPUTER

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by

J. Leung* and H. Liu**

ABSTRACT

Many experimental situations require the examination of the effects of varying two or more [illegible]s. When the experiment involves complex interrelationships between m[illegible] diverse factors, the classical one-variable-at-a-time method of designing a research program is not adequate to provide reliable information and is quite time-consuming. Factorial designs minimize the chance of faulty experiment design, embody concrete procedures for testing the validity and reliability of the test data and for indicating mandatory changes in the design, and enable the researcher to obtain meaningful information with the least number of tests. A conversational program FADESN written in FORTRAN is described in this paper to find the optimum conditions rapidly and accurately with simple statistical evaluation. Five designs of different numbers of levels and observations, that are generally used, are presented in this program. Regression coefficients from the response equation are also calculated with simplified least squares method.

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UNE ANALYSE DES RÉSULTATS DES EXPÉRIENCES FACTORIELLES PROGRAMMÉES PORTANT SUR LE TRAITEMENT DES MINÉRAUX

par

J. Leung* and H. Liu**

RÉSUMÉ

Dans plusieurs situations expérimentales, il est nécessaire d'examiner les résultats produits par le changement de deux facteurs ou plus. Quand l'expérience comprend des rapports complexes entre de nombreux facteurs différents, la méthode classique d'utiliser un variable à la fois pour concevoir un programme de recherche ne peut fournir des renseignements sûrs et exige beaucoup de temps. Les modèles factoriels limitent les possibilités de conceptions expérimentales inexactes, renferment des procédures concrètes permettant de justifier la justesse et la fiabilité des données sur les essais, servent à indiquer les modifications qui doivent être apportées au modèle et permettent au chercheur d'obtenir des renseignements importants tout en exigeant un nombre minimal d'essais. Le rapport présente un programme conversationnel FADESN en language Fortran conçu dans le but de découvrir les conditions optimales avec rapidité et exactitude au moyen d'une évaluation statistique simple. Ce programme comprend cinq modèles des différents niveaux et critères d'observation en usage. Les coefficients de régression de l'équation de la réponse sont également calculés à l'aide de la méthode des moindres carrés simplifiée.

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1. PROGRAM IDENTIFICATION

Program Title: FActorial DESigN Analysis

Program Code Name: FADESN

Author: J. Leung

Organization: Energy, Mines and Resources, Canada Centre for Mineral and Energy Technology (CANMET), Mineral Processing Laboratory

Date: September 1984

Updates: None, this is version 0.

Source Language: CDC FORTRAN extended 4.8, complying with American National Standards Institute FORTRAN language as described in X3.9 - 1966.

Availability: Source listing available from author at CANMET.

2. ENGINEERING DOCUMENTATION

2.1 NARRATIVE DESCRIPTION

2.1.1 Introduction

Experiments carried out by metallurgists, engineers and scientists, whether in the laboratory or in the plant, are in general intended to determine the effect of one or more factors on the recovery or quality of a product, the performance of a piece of equipment, the optimization of experimental conditions, the power or fuel consumption of a process, etc. Factorial design is applied in order that the effect of changing any one variable or factor can be efficiently assessed independently of the others. To achieve this, one has to decide on a set of values or levels, for each of the factors to be studied, and to carry out one or more trials of the process with each of the possible combinations (these being the treatments or treatment combinations) of the levels of the factors. The number of trials can be restricted in certain well-defined ways (1).

FADESN is written to analyze the results of factorial experiments in a few seconds of time. It calculates the sum of squares, regression coefficients, the experimental error (either from replicate measurements or from interaction effects that are insignificant and negligible) and finally performs the F-test to determine the significance of each effect.

2.1.2 Fractional Factorial Design

When the number of factors is large, a complete (full) factorial experiment, in which all possible combinations of all the levels of the different factors are investigated, will involve a large number of tests. For example, an investigation of five factors each at two levels will entail $2^5 = 32$ observations. It is probably not practical and economical to carry out all 32 observations. Therefore, a researcher with prior knowledge of the process or similar processes, may only investigate the main effects of the factors and their more important interactions in a fraction of the complete tests. In the above example, he may only carry out half-replicate, i.e., 16 tests or even quarter-replicate, i.e., 8 tests by neglecting the higher-order interactions which are negligible and unimportant. Even when an experiment of large magnitude may eventually be required, he would prefer to carry out the work as a series of smaller experiments and review the situation during the course of

the process. The construction of fractional factorial designs can be found in other sources (1,2,3,4) and will not be repeated here.

2.1.3 Select a Factorial Design

Usually two-level factorial designs are used to explore a region only when the main effects of the factors are linear or approximately linear over the region and interaction effects are relatively small compared to the main effects. However, when main effects are not linear over the region of interest, more than two levels for the factors are necessary for a complete study of the response surface. If the factors are measurable (quantitative, e.g., temperature, amount of reagent time of reaction), the experiment leads to a curve expressing the results in terms of the levels of the factors (see Appendix A).

This program includes the five most commonly used designs $L_4(2^3)$*, $L_8(2^7)$, $L_{16}(2^{15})$, $L_9(3^4)$ and $L_{16}(4^5)$ (Table 1) in multifactorial experiment at two, three or four levels (5). Before selecting a factorial design, one has to decide the numbers of factors, levels and observations; as well as whether replicate measurements are required. With acquired knowledge and experience, one may be able to estimate the experimental error either from prior experiments or higher-order interaction effects which are known to be insignificant (2.1.5). The user can either enter his own design matrix or use the one in the program. When one of the designs has been chosen from Table 1, the user has to adjust his treatment combinations or observations according to the table.

2.1.3.1 *2-level*

For easy computation, the two levels of a factor are coded to give values of -1 and +1 (-1 for lower level and +1 for higher level, Ref. 6). The possible combinations for $L_4(2^3)$, $L_8(2^7)$ and $L_{16}(2^{15})$ are shown in Tables 2a, b, and c. Each column represents an effect (either from main factor or interaction) and for N tests, the maximum factors that can be applied are (N-1). Therefore $L_4(2^3)$ can be for two or at most three factors (Table 2a).

---

*$L_4(2^3)$ means a design matrix (or information matrix) with dimensions 4x3 (rows x columns); all factors at 2-level. Four and three are the numbers of observations and effects respectively.

In general, $L_8(2^7)$ is most frequently used in two-level fractional factorial experiment. For example, for a two-level, four-factor experiment, for the same amount of work, it is better to use the half-replicate $L_8(2^7)$ (eight observations with each repeated once) than to use the 2^4 complete (full) factorial design $L_{16}(2^{15})$, because $L_8(2^7)$ provides more weight on the main effects. This method of sacrificing the interaction effects to improve the accuracy of the main effects is often used by engineers in mineral processing research.

With eight observations, $L_8(2^7)$ (Table 2b) may therefore be used to estimate the effects of:

i) three factors and all interactions between them. This constitutes a complete 2^3 factorial design;
ii) four factors and the first degree (two-factor) interactions between three of them, all other interactions being assumed zero or negligible;
iii) five factors and an interaction of one factor with each of two others, all other interactions assumed zero or negligible;
iv) six factors and one first degree interaction, all other interactions assumed zero or negligible;
v) seven factors, all interactions assumed zero or negligible. This is called "saturated design". If any of the interactions is significant, the reliability of the result will be affected.

With sixteen observations, $L_{16}(2^{15})$ (Table 2c) may be used to estimate the effects of:

i) four factors and all interactions between them. This constitutes a complete 2^4 factorial design;
ii) five factors and all first degree interactions on the assumption that all second degree (three-factor) and higher-order interactions are zero or negligible;
iii) six factors and at most seven first degree interactions (between four of the factors) all other interactions assumed negligible. Column 7 and 14 are "empty" and can be for error estimation (2.1.5);

iv) seven factors and at most seven first degree interactions (between four of the factors), all other interactions assumed negligible. Column 7 is "empty" and can be used for error estimation;

v) eight or more factors with appropriate restrictions on the allowable first degree interactions.

Each column in Tables 2a, b, and c has one degree of freedom, i.e., for a design with n factors, p levels per factor, the p^n-1 degrees of freedom between the p^n treatment combinations may be partitioned into $\frac{p^n-1}{p-1}$ sets of (p-1) degrees of freedom (4).

2.1.3.2 *3-level*

The basic principles of fractional replicates of 3^n designs are similar to those of 2^n designs, but the construction of the designs and their interpretation is much more complicated. In practice, fractional replication for three levels is best avoided whenever there is any likelihood of several appreciable first degree (two-factor) interactions and whenever the researcher has sufficient freedom of choice in the levels to be examined. When the factors are quantitative, two levels per factor should be the rule when fractional replication is contemplated. For <u>qualitative</u> factors (i.e., unmeasurable) it may sometimes be necessary to use three levels.

A design with nine observations $L_9(3^4)$ (Table 2d) is chosen for this purpose and it may be used to estimate the effects of:

i) two factors and all interactions between them. This constitutes a complete 3^2 factorial design;

ii) two factors on the assumption that there is no interaction between them. Four degrees of freedom are left over for error estimation;

iii) three factors on the assumption that all interactions are negligible. Two degrees of freedom are left over for error estimation;

iv) four factors and all interactions being assumed negligible. No degrees of freedom are left over for error estimation. This is called "saturated" design.

Each column in Table 2d has two degrees of freedom. For easy computation, the three levels are coded to give values of -1, 0, +1 at equal intervals (-1 for the lower level, 0 for the middle level and +1 for the higher level). Comparing with quantitative factors, these values represent the linear component of the effect (7, Appendix A).

2.1.3.3 *4-level*

As was stated regarding three levels, fractional application for four levels is best avoided whenever possible. For a quantitative factor, the four levels imply an extra component, i.e., the cubic term in the response curve (Appendix A). A design with sixteen observations $L_{16}(4^5)$ is chosen for qualitative factor in this program and it may be used to estimate the effects of:

i) two factors and all interactions between them. This constitutes a complete 4^2 factorial design;
ii) two factors on the assumption that no interaction occurs between them. Nine degrees of freedom are left over for error estimation;
iii) three factors on the assumption that all interactions are negligible. Six degrees of freedom are left over for error estimation;
iv) four factors on the assumption that all interactions are negligible. Three degrees of freedom are left over for error estimation;
v) five factors and all interactions being assumed negligible. No degrees of freedom are left over for error estimation.

For easy computation, the four levels are coded to give values of -3, -1, +1, +3 at equal intervals. These values are the linear components for quantitative factors. Each column in this design has three degrees of freedom.

2.1.4 Orthogonality

The property of the factorial designs that enables the experiment symmetrical in all the factors by testing all possible treatment combinations

the same number of times, thus simplifying both the calculations and the interpretation of the results, in addition to giving maximum efficiency, is known as orthogonality. Orthogonality ensures that all the main effects and interactions can be independently estimated without entanglement.

Consider two linear functions of the observation $y_1, y_2, \ldots y_n$:

$$E_1 = a_1y_1 + a_2y_2 + \ldots + a_ny_n$$
$$E_2 = b_1y_1 + b_2y_2 + \ldots + b_ny_n$$

The necessary and sufficient condition for the two linear functions to be orthogonal to each other is:

$$\Sigma ab = a_1b_1 + a_2b_2 + \ldots + a_nb_n = 0 \qquad (1)$$

The effect functions used in the analysis of factorial experiments are all orthogonal to one another and they are also subject to another two conditions:

$$\Sigma a = \Sigma b = 0$$
$$\Sigma a^2 = \Sigma b^2$$

However, these two conditions are not necessary for orthogonality. For illustration, the user may see that all columns in Table 1 are orthogonal to each other.

2.1.5 Experimental Error

Experimental error usually originates from several sources and with suitable experimental designs may be divided into components corresponding to their origin. Sometimes a suitable estimate of the experimental error exists from previous records or experience. When this estimation is not suitable, the experiment may be designed so that some of the higher order interactions that are known to be nonexistant or statistically unimportant will provide this value when the data are analyzed. (Usually their sums of squares are relatively small). If all possible interactions are known to exist or so few degrees of freedom are available for error estimation such that the tests for significance is not sensitive, it may be necessary to make two or more replications of some of the tests. Whenever an estimate of experimental error is

obtained from an experiment, some consideration should be given to the possibility of combining it with previous estimates to get a more precise estimate.

For illustration, in the design $L_8(2^7)$ (Table 2b) with three factors A, B and C, the combined sum of squares from columns five, six and seven with a total of three degrees of freedom (one from each column) can be used to estimate the variance of the experimental error provided that the first degree interactions AC and BC and second degree interaction ABC are negligible. Also in design $L_{16}(2^{15})$ (Table 2c), with four factors A, B, C and D, columns seven, eleven, thirteen, fourteen and fifteen (assigned for second degree interactions ABC, ABD, ACD, BCD and third degree interaction ABCD respectively) can be used for the same purpose with a total of five degrees of freedom, provided that these higher order interactions are small enough to be ignored. The "empty" columns in the same table are also based on the same assumption and can be used for error estimation.

2.2 METHODS OF SOLUTION

The algorithms used in FADESN for factorial designs are as follows:

2.2.1 Regression Coefficients

The method of evaluation for the experimental design aims to relate the quantity measured, i.e., the observation, to the variables of the design, i.e., the factors. This dependence is usually expressed in the form of a regression equation:

$$Y = b_0 + b_1X_1 + \ldots + b_jX_j + \ldots + b_mX_m$$

where Y = response of observation
b = regression coefficients
X = factor or interaction effects
m = number of effects

Since the linear functions of the n observations are made orthogonal, each regression coefficient can be determined very simply with Least Squares method (1):

$$b_j = \frac{\sum_{i=1}^{n} Y_i X_{ij}}{\sum_{i=1}^{n} X_{ij}^2} \qquad \text{for jth coefficient, } j=1,\ldots,m$$

$$b_0 = \frac{\sum_{i=1}^{n} Y_i}{n} \qquad \text{intercept}$$

For factors with more than two levels, the derivation by regression coefficients is shown in Appendix A and the interpretation for interaction effects is more complicated and physically uncertain. The user should therefore use the algorithm with care.

2.2.2 Sum of Squares

$$SS_j = \frac{\left(\sum_{i=1}^{n} Y_i X_{ij}\right)^2}{\sum_{i=1}^{n} X_{ij}^2} \qquad \text{for jth coefficient, } j=1,\ldots,m$$

$$= b_j^2 \cdot \sum_{i=1}^{n} X_{ij}^2$$

SST = total sum of squares

$$= \sum_{i=1}^{n} Y_i^2 - \frac{\left(\sum_{i=1}^{n} Y_i\right)^2}{n}$$

$$= \sum_{i=1}^{n} Y_i^2 - \frac{Y_T^2}{n}$$

where Y_T = sum of n observations.

2.2.3 Mean Sum of Squares

$$AMS_j = \frac{SS_j}{\phi_j} \qquad \text{for } j^{th} \text{ coefficient}$$

where ϕ_j = number of degrees of freedom for b_j.

2.2.4 Degrees of Freedom

$$NDF = P-1 \qquad \text{for each coefficient}$$

where P = number of levels per factor (same for all factors).

$$NDFT = \text{total number of degrees of freedom} = n-1$$

where n = number of observations.

2.2.5 Experimental Error from Repeat Measurements

$$VAR = \frac{\sum_{i=1}^{m} V_i}{\sum_{i=1}^{m} \phi_i}$$

where V_i = error variance for i^{th} repeat set

$$= \sum_{j=1}^{P_i} (Y_{ij} - \bar{Y}_i)^2$$

$\bar{Y}_i$ = average of Y_{ij} for i^{th} repeat set

$$= \frac{\sum_{j=1}^{P_i} Y_{ij}}{P_i}$$

P_i = number of replication for i^{th} repeat set

m = number of repeat sets

ϕ_i = number of degrees of freedom for i^{th} repeat set

$= P_i - 1$

2.2.6 Experimental Error from Insignificant Interactions

$$VAR = \frac{SS_e}{\phi_e}$$

$$= \frac{SST - \Sigma SS_j}{\phi_T - \Sigma\phi_j}$$

where SSe = combined sum of squares of insignificant interactions or remaining interactions

ϕ_e = combined number of degrees of freedom of SSe

SST = total sum of squares

ϕ_T = total number of degrees of freedom

= n-1

ΣSS_j = combined sum of squares of important effects

$\Sigma\phi_j$ = combined number of degrees of freedom of ΣSS_j

The value SSe provides the sum of squares about regression, i.e., the sum of squares of the discrepanices between the observed values and those values predicted by the regression equation. Therefore it also includes all the higher order interactions which may not originally have been included in the regression model. If some of these higher order interactions are significant and yet have been excluded from the equation, then SSe will be inflated by these missing terms (see 2.8.3, sample run 3).

2.2.7 F-test

$$FT_j = \frac{AMS_j}{VAR} \qquad \text{for jth coefficient}$$

An effect is said to be significant, if its F-ratio FT is greater than FDIS (ϕ_1, ϕ_2) which is taken from the table of F-distribution and depends on the confidence level and ϕ_1, ϕ_2, the numbers of degrees of freedom of AMS and VAR respectively. In mineral processing research, a 90% confidence level is usually enough, however, for critical experiments, 95% confidence level is required.

FDIS is very sensitive to ϕ_2 when ϕ_2 is small. For example, when $\phi_2 = 1$, with 95% confidence level and $\phi_1 = 1$, FDIS (1,1) = 161.4 (Table 3). FT of an effect has to be greater than 161 before the effect is considered to be significant. When $\phi_2 = 2$, FDIS (1,2) = 18.5 and when $\phi_2 = 3$, FDIS (1,3) = 10.1. However, if the error variance VAR is exactly known, (i.e., $\phi_2 = \infty$) the value FDIS will only be 3.8. Therefore, the user is recommended either to perform more replicate tests or to include more "empty" columns or insignificant higher order interactions for better representative estimation of error variance.

2.3 PROGRAM CAPABILITIES

FADESN enables the user to enter up to 32 observations and 16 effects (including main factors and interactions). If more numbers are needed, DIMENSION statements in the program have to be modified accordingly. The F-distribution tables (with 90% and 95% confidence level respectively) are provided with 3 and 9 respectively as the maximum numbers of degrees of freedom for numerator and denominator. Up to four levels per factor are allowed.

2.4 DATA INPUT

All data are entered in free-field format with blank, comma or carriage return as separators. If there are replicate measurements for each test, the user is recommended to input only its mean value as test result. All the replicate results are required in the latter part of the program and are used to calculate the experimental error.

2.5 PROGRAM OPTIONS

The user is allowed either to use one of the designs $L_4(2^3)$, $L_8(2^7)$, $L_{16}(2^{15})$, $L_9(3^4)$ and $L_{16}(4^5)$ provided in the program or enter his own. When there are no replicate measurements from which the error variance can be derived, or when the estimation of this value cannot be made, the program will print out the sum of squares and the number of degrees of freedom for each effect (column). The user then has the opportunity to enter the column numbers that he thinks are unimportant and can be used to estimate the error variance.

2.6 PRINTED OUTPUT

See sample runs outputs in Appendix B.

2.7 FLOWCHART

The general flowchart of the program modules is given in Fig. 1.

2.8 SAMPLE RUNS

This section presents a few sample runs to show how FADESN can be applied in the mineral processing laboratory. The detailed runs are illustrated in Appendix B.

2.8.1 Sample Run 1 (6)

A major porphyry copper mine in British Columbia wanted to improve the performance PW (present worth) of its grinding circuit. From a preliminary survey the three variables believed to have the most effect were identified and each variable was set at two levels:

Variable	Low	High
F (rod mill feed rate, stph)	320	340
W (pump box water addition rate, USGPM)	1000	1150
A (cyclone apex size, in.)	6	7

The variables can be coded as:

Variable	Low	High
F*	-1	+1
W*	-1	+1
A*	-1	+1

To include all the interaction effects, the regression equation can be written as:

$$PW\ (\$ \times 10^{-6}) = b_0 + b_1F^* + b_2W^* + b_3A^* + b_4F^*W^* + b_5F^*A^* + b_6\ W^*A^* + b_7\ F^*W^*A^*$$

All combinations of levels were tested giving a complete 2^3 factorial design plus replicate tests at the midpoint of the design (i.e., at F = 330 stph, F* = 0; W = 1075 USGPM, W* = 0; A = 6.5 in., A* = 0). The tests are:

Test No.	F*	W*	A*	F*W*	F*A*	W*A*	F*W*A*	PW ($ x 10^{-6})
1	-1	-1	-1	+1	+1	+1	-1	35.522
2	+1	-1	-1	-1	-1	+1	+1	36.184
3	-1	+1	-1	-1	+1	-1	+1	35.921
4	+1	+1	-1	+1	-1	-1	-1	37.125
5	-1	-1	+1	+1	-1	-1	+1	35.599
6	+1	-1	+1	-1	+1	-1	-1	36.106
7	-1	+1	+1	-1	-1	+1	-1	35.46
8	+1	+1	+1	+1	+1	+1	+1	37.433
9	0	0	0	0	0	0	0	36.334
10								36.759
11								36.616
12								36.227

The problem is to find the variables that have significant effect on PW.

Discussion of Sample Run 1

The user can either enter the treatments manually column by column (n = 8) or use design two (Tables 1b and 2b) in the program. In the latter case, the order of the effects A* and F*A* is reversed. The user should beware of their corresponding regression coefficients.

The experimental error calculated from the 4 replicate tests is 0.0605 with 3 degrees of freedom. Over the range tested, only ore feed and water feed rate have a significant effect on PW while the apex/diameter and all interaction effects do not. The regression equation at 95% confidence level is:

$$PW = 36.169 + 0.543\ (F^*) + 0.316\ (W^*)$$

The apex can be eliminated from any future work and the ore and water feed rates increased to the +1 level and a new series of experiments designed in order to determine optimum settings.

2.8.2 Sample Run 2 (5)

A copper mine used copper sulphate and oxalic acid as activators to

separate chalcopyrite from pyrite. In order to determine the best amount of the agent to be used, an experiment was designed to investigate. Each agent was set at three levels:

Agent	Low	Medium	High
A (oxalic acid, gm/t)	1000	1500	2000
B (copper sulphate, gm/t)	180	200	220

The codes are -1, 0, +1 respectively and the efficiences of the tests using the qualitative design $L_9(3^4)$ for simple illustration are:

Test No.	1(A)	2(B)	3	4	Efficiency (%)
1	-1	-1	-1	-1	41.3
2	-1	0	0	0	42.8
3	-1	+1	+1	+1	43.0
4	0	-1	0	+1	43.5
5	0	0	+1	-1	44.9
6	0	+1	-1	0	44.7
7	+1	-1	+1	0	46.0
8	+1	0	-1	+1	45.0
9	+1	+1	0	-1	45.9

(Actually, both factors are measurable (i.e., quantitative) and each has three levels. A quadratic function is implied in the response equation. The example using orthogonal polynomial method is given in Appendix A with output in Appendix B. Both qualitative and quantitative methods give the same result - therefore it is recommended to use the former method whenever possible).

No replicate measurements were arranged because previous experience indicated that there was no interaction between the two agents and the columns 3 and 4 could be used to estimate the variance of the experimental error.

Discussion of Sample Run 2

Design four from the program was used and since the error variance was not known, none was entered. The sums of squares and the numbers of the degrees of freedom of columns 3 and 4 gave the error variance of 0.6106 (with 4 degrees of freedom).

The results showed that the effect of oxalic acid is significant and the higher level (2000 gm/t) produced best efficiency (tests 7, 8 and 9) while the effect of copper sulphate is not significant in this region and the lower level (180 gm/t) could be used.

2.8.3 Sample Run 3 (5)

A flotation experiment to separate feldspar from quartz in a silicate sand ore was designed. There were five factors each at four levels:

Factor	Level 1	Level 2	Level 3	Level 4
A (sulphuric acid, gm/t)	1000	1500	2000	3000
B (sodium fluoride, gm/t)	1000	2000	3000	4000
C (diesel fuel, gm/t)	600	750	900	1200
D (combined ammonia, gm/t)	600	750	900	1200
E (pine oil, gm/t)	200	250	300	400

The levels were coded as -3, -1, +1, +3 respectively even though the levels are not equally spaced and design $L_{16}(4^5)$ was used for primitive experiment with test results as:

Test No.	1(A)	2(B)	3(C)	4(D)	5(E)	Efficiency (%)
1	-3	-3	-3	-3	-3	56
2	-3	-1	-1	-1	-1	52
3	-3	+1	+1	+1	+1	68
4	-3	+3	+3	+3	+3	9
5	-1	-3	-1	+1	+3	75
6	-1	-1	-3	+3	+1	65
7	-1	+1	+3	-3	-1	38
8	-1	+3	+1	-1	-3	50
9	+1	-3	+1	+3	-1	61
10	+1	-1	+3	+1	-3	77
11	+1	+1	-3	-1	+3	55
12	+1	+3	-1	-3	+1	63
13	+3	-3	+3	-1	+1	73
14	+3	-1	+1	-3	+3	49
15	+3	+1	-1	+3	-3	82
16	+3	+3	-3	+1	-1	66

Discussion of Sample Run 3

This is an example of "saturated" design. The analytical results showed that at 90% confidence level none of the factors had significant effect. The error variance (738.763) was estimated from column 4 which had the smallest sum of squares (99.012) by subtracting the combined sum of squares (from columns 1, 2, 3, and 5) from the total sum of squares. The error variance had been inflated which indicated that there were other effects which were significant and yet had not been taken into consideration (i.e., the interaction effects between the factors).

From the tests, one noticed that tests 5, 10, 13 and 15 yielded the best efficiencies (75%, 77%, 73% and 82%) and they all had one common property, i.e., consumption on reagent B was less than on A; while in tests where the situation reversed, the efficiences dropped especially in test 4 (9%) where reagent B was used four times more than the reagent A. Therefore the interaction between A and B had a very significant effect which was impossible to uncover by this fractional factorial design $L_{16}(4^5)$. A further complete experiment design specifically for these two factors is required.

This example indicates that when the interaction effects exist or the design reaches "saturation", it is recommended that 2-level design be used to avoid confused interpretation.

3. SYSTEM DOCUMENTATION

3.1 COMPUTER EQUIPMENT

FADESN was developed on a CYBER 730/30 under NOS/BE, Energy, Mines and Resources. A time-sharing terminal was used for the conversational capabilities of the program.

3.2 VARIABLES AND SUBROUTINES

A list of variables is given in Table 4. There are 5 routines in the program. Their purpose and usage are contained in the prologue in the beginning of each routine.

3.3 DATA STRUCTURES

All data is normally read on logical unit 5 and printed in logical unit 6.

3.4 STORAGE REQUIREMENTS

The program requires about 60 K words of memory for execution.

4. OPERATING DOCUMENTATION

4.1 OPERATOR INSTRUCTIONS

Compilation, loading and execution of this stand-alone package are done under the normal rules of the operating system.

4.2 OPERATING MESSAGES

None besides the standard operating system messages.

4.3 CONTROL CARDS

In most instances, typing the program name after accessing the compiled version will execute the job.

4.4 ERROR RECOVERY

Program must be restarted on error.

4.5 RUN TIME

It most instances, a fraction of a second of CP time is sufficient to produce a solution.

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Table 1a - Design matrix $L_4(2^3)$*

	Column No.		
Test No.	1	2	3
1	-1	-1	+1
2	+1	-1	-1
3	-1	+1	-1
4	+1	+1	+1

*Design for 4 observations, 3 "effect" columns, all factors at 2 levels.

Table 1b - Design matrix $L_8(2^7)$

	Column No.						
Test No.	1	2	3	4	5	6	7
1	-1	-1	+1	-1	+1	+1	-1
2	+1	-1	-1	-1	-1	+1	+1
3	-1	+1	-1	-1	+1	-1	+1
4	+1	+1	+1	-1	-1	-1	-1
5	-1	-1	+1	+1	-1	-1	+1
6	+1	-1	-1	+1	+1	-1	-1
7	-1	+1	-1	+1	-1	+1	-1
8	+1	+1	+1	+1	+1	+1	+1

Table 1c - Design matrix $L_{16}(2^{15})$

	Column No.														
Test No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	-1	-1	+1	-1	+1	+1	-1	-1	+1	+1	-1	+1	-1	-1	+1
2	+1	-1	-1	-1	-1	+1	+1	-1	-1	+1	+1	+1	+1	-1	-1
3	-1	+1	-1	-1	+1	-1	+1	-1	+1	-1	+1	+1	-1	+1	-1
4	+1	+1	+1	-1	-1	-1	-1	-1	-1	-1	-1	+1	+1	+1	+1
5	-1	-1	+1	+1	-1	-1	+1	-1	+1	+1	-1	-1	+1	+1	-1
6	+1	-1	-1	+1	+1	-1	-1	-1	-1	+1	+1	-1	-1	+1	+1
7	-1	+1	-1	+1	-1	+1	-1	-1	+1	-1	+1	-1	+1	-1	+1
8	+1	+1	+1	+1	+1	+1	+1	-1	-1	-1	-1	-1	-1	-1	-1
9	-1	-1	+1	-1	+1	+1	-1	+1	-1	-1	+1	-1	+1	+1	-1
10	+1	-1	-1	-1	-1	+1	+1	+1	+1	-1	-1	-1	-1	+1	+1
11	-1	+1	-1	-1	+1	-1	+1	+1	-1	+1	-1	-1	+1	-1	+1
12	+1	+1	+1	-1	-1	-1	-1	+1	+1	+1	+1	-1	-1	-1	-1
13	-1	-1	+1	+1	-1	-1	+1	+1	-1	-1	+1	+1	-1	-1	+1
14	+1	-1	-1	+1	+1	-1	-1	+1	+1	-1	-1	+1	+1	-1	-1
15	-1	+1	-1	+1	-1	+1	-1	+1	-1	+1	-1	+1	-1	+1	-1
16	+1	+1	+1	+1	+1	+1	+1	+1	+1	+1	+1	+1	+1	+1	+1

Table 1d - Design matrix $L_9(3^4)$

	Column No.			
Test No.	1	2	3	4
1	-1	-1	-1	-1
2	-1	0	0	0
3	-1	+1	+1	+1
4	0	-1	0	+1
5	0	0	+1	-1
6	0	+1	-1	0
7	+1	-1	+1	0
8	+1	0	-1	+1
9	+1	+1	0	-1

Table 1e - Design matrix $L_{16}(4^5)$

	Column No.				
Test No.	1	2	3	4	5
1	-3	-3	-3	-3	-3
2	-3	-1	-1	-1	-1
3	-3	+1	+1	+1	+1
4	-3	+3	+3	+3	+3
5	-1	-3	-1	+1	+3
6	-1	-1	-3	+3	+1
7	-1	+1	+3	-3	-1
8	-1	+3	+1	-1	-3
9	+1	-3	+1	+3	-1
10	+1	-1	+3	+1	-3
11	+1	+1	-3	-1	+3
12	+1	+3	-1	-3	+1
13	+3	-3	+3	-1	+1
14	+3	-1	+1	-3	+3
15	+3	+1	-1	+3	-3
16	+3	+3	-3	+1	-1

Table 2a - Various designs of $L_4(2^3)$ with factors A, B,...,etc.

	Column No.		
Factor No.	1	2	3
2	A	B	AB*
3	A	B	C

*AB - First degree interaction between factors A and B

Table 2b - Various designs of $L_8(2^7)$ with factors A,B,..,etc.

Factor No.	Column No. 1	2	3	4	5	6	7
3	A	B	AB	C	AC	BC	ABC*
4	A	B	AB	C	AC	BC	D
			CD		BD	AD	
5	A	B	AB	C	AC	D	E
	DE	CD	CE	BD	BE	BC AE	AD
6	A	B	AB	C	D	E	F
	CD EF	CE DF	CF DE	AD BE	AC BF	AF BC	AE BD
7	A BC	B AC	C AB	D AE	E AD	F AG	G AF
	DE FG	DF EG	DG EF	BF CG	BG CF	BD CE	BE CD

*ABC - Second degree interaction from factors A, B and C

Table 2c - Various designs of $L_{16}(2^{15})$ with factors A,B,...,etc.

Factor No.	Column No. 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
4	A	B	AB	C	AC	BC	ABC	D	AD	BD	ABD	CD	ACD	BCD	ABCD**
5	A	B	AB	C	AC	BC	DE	D	AD	BD	CE	CD	BE	AE	E
	A	B	AB	C	AC	BC		D	AD	BD	E	CD	F		CE
6			DE		DF	EF			BE	AE		AF			BF
									CF						
	A	B	AB	C	AC	BC		D	AD	BD	E	CD	F	G	CE
7			DE		DF	EF			BE	AE		AF			BF
			FG		EG	DG			CF	CG		BG			AG
	A	B	AB	C	AC	BC	H	D	AD	BD	E	CD	F	G	CE
8			DE		DF	EF			BE	AE		AF			BF
			FG		EG	DG			CF	CG		BG			AG
			CH		BH	AH			GH	FH		EH			DH

**ABCD - Third degree interaction from factors A, B, C and D

Table 2d - Various designs of $L_9(3^4)$ with factors A,B,...,etc.

	Column No.			
Factor No.	1	2	3	4
2	A	B	I(AB)*	J(AB)**
3	A	B	C	J(AB)
4	A	B	C	D

*I(AB) - I-interactions between A and B

**J(AB) - J-interactions between A and B

Each column has two degrees of freedom

(Ref. 1,4)

Table 3 - F-distribution

Confidence level	0.90			0.95		
ϕ_2 \ ϕ_1	1	2	3	1	2	3
1	39.86	49.5	53.59	161.4	199.5	215.7
2	8.53	9.0	9.16	18.51	19.0	19.16
3	5.54	5.46	5.39	10.13	9.55	9.28
4	4.54	4.32	4.19	7.71	6.94	6.59
5	4.06	3.78	3.62	6.61	5.79	5.41
6	3.78	3.46	3.29	5.99	5.14	4.76
7	3.59	3.26	3.07	5.59	4.74	4.35
8	3.46	3.11	2.92	5.32	4.46	4.07
9	3.36	3.01	2.81	5.12	4.26	3.86
10	3.28	2.92	2.73	4.96	4.10	3.71

ϕ_1: number of degrees of freedom of numerator

ϕ_2: number of degrees of freedom of denominator

Table 4 - Variables in FADESN

Name	Dimension	Description
N	1	Number of tests or observations
M	1	Number of factors and interactions
P	1	Number of levels per factor
X	N*M	Design matrix (treatment combinations)
Y	N	Test results or responses
B	M+1	Regression coefficients
SS	M+1	Sum of squares
AMS	M+1	Mean sum of squares
FT	M+1	F-ratio (AMS/VAR)
VAR	1	Variance of experimental error
NVAR	1	Number of degrees of freedom of VAR
NDF	1	Number of degrees of freedom of each effect

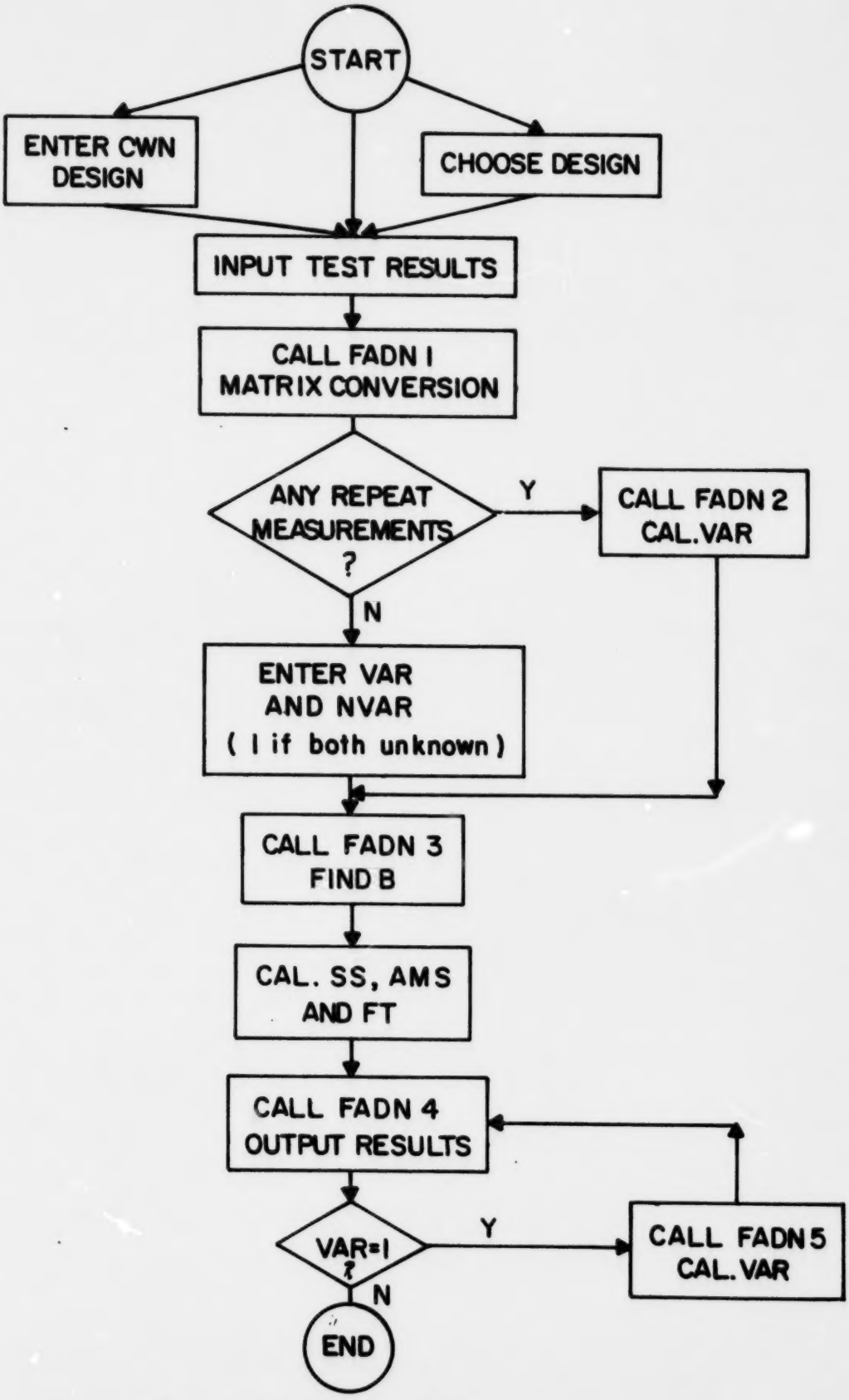

Fig. 1 - Flowchart of FADESN

APPENDIX A

ORTHOGONAL POLYNOMIALS FOR QUANTITATIVE FACTOR

ORTHOGONAL POLYNOMIALS FOR QUANTITATIVE FACTOR

When the factors are measurable, the experiment leads to a curve expressing the results in terms of the levels of the factors. Over a finite range of the factors, assuming there are two factors A and B, the response surface is represented by means of a polynomial e.g.,

$$y = a_0 + a_1 x + a_{11} x^2 + \ldots$$

where x denotes the level of the factor A and y the response. When three levels are used, a quadratic function is implied; when four levels are used, a cubic function, and so on. The coefficients a_0, a_1,..., are not mutually orthogonal, but it is possible to transform the polynomial into another polynomial:

$$y = b_0 + b_1 X_1 + b_2 X_2 + \ldots \qquad \text{A-1}$$

in which the constants b_0, b_1,... are now mutually orthogonal linear functions of the observations and X_1, X_2,... are respectively linear, quadratic, etc., functions of the variable x. For values of x spaced at equal intervals, the function is given by (ref. 7 in main text):

$$X_1 = x - \bar{x} \qquad \text{linear}$$

$$X_2 = (x - \bar{x})^2 - \frac{(n^2 - 1)}{12} \qquad \text{quadratic}$$

$$X_3 = (x - \bar{x})^3 - \frac{(3n^2 - 7)}{20}(x - \bar{x}) \qquad \text{cubic}$$

etc.,

where n is the number of levels.

The above relationships apply to the levels of other factors B, C,... etc. The orthogonality implies that, over N observed X-values:

$$\sum_{i=1}^{N} (X_j)_i \, (X_k)_i = 0$$

for all j and k which are not equal. To make the above values X_1, X_2,... integral, the following transformation is introduced:

$$X'_j = \lambda \, X_j$$

where λ = scaling factor.

The X'-values for the linear, quadratic and cubic components are shown in the following table:

3-level		4-level		
linear X'_1	Quadratic X'_2	linear X'_1	Quadratic X'_2	Cubic X'_3
-1	+1	-3	+1	-1
0	-2	-1	-1	+3
		+1	-1	-3
+1	+1	+3	+1	+1
λ 1	3	2	1	$\frac{10}{3}$

The coefficients in the regression equation are:

$$b_0 = \frac{\sum_{i=1}^{N} Y_i}{N}$$

where N = number of observations

$$b_j = \frac{\sum_{i=1}^{N} Y_i (X_j)_i}{\sum_{i=1}^{N} (X_j)_i^2}$$

$$= \frac{\lambda \sum_{i=1}^{N} Y_i (X'_j)_i}{\sum_{i=1}^{N} (X'_j)_i^2}$$

The sum of squares of b_j is

$$SS_j = \frac{\left[\sum_{i=1}^{N} Y_i (X_j)_i\right]^2}{\sum_{i=1}^{N} (X_j)_i^2}$$

$$= \frac{\left[\sum_{i=1}^{N} Y_i (X_j')_i\right]^2}{\sum_{i=1}^{N} (X_j')_i^2}$$

With the experimental error derived from other sources (2.2.5, 2.2.6), the F-test can be performed to test the significance of each coefficient. Substituting the important coefficients and the X-values in terms of the levels x, in Eq 1, the regression equation can be obtained.

For illustration, sample run 2 (2.8.2) is used here as an example. Both two factors oxalic acid and copper sulphate are measurable and each has three levels. Therefore their effects can be represented by the polynomial expression and their numbers of degrees of freedom are:

	Effect	Components	Degrees of freedom	
Main effects:	A	linear	1	2
		quadratic	1	
	B	linear	1	2
		quadratic	1	
Interactions:	AB	linear A and linear B	1	4
		linear A and quadratic B	1	
		quadratic A and linear B	1	
		quadratic A and quadratic B	1	

The orthogonal polynomials for the effects are shown in the following table:

	Main effect				Interaction			
Test No.	A(L)*	A(Q)**	B(L)	B(Q)	A(L)B(L)	A(L)B(Q)	A(Q)B(L)	A(Q)B(Q)
1	-1	+1	-1	+1	+1	-1	-1	+1
2	-1	+1	0	-2	0	+2	0	-2
3	-1	+1	+1	+1	-1	-1	+1	+1
4	0	-2	-1	+1	0	0	+2	-2
5	0	-2	0	-2	0	0	0	+4
6	0	-2	+1	+1	0	0	-2	-2
7	+1	+1	-1	+1	-1	+1	-1	+1
8	+1	+1	0	-2	0	-2	0	-2
9	+1	+1	+1	+1	+1	+1	+1	+1

*A(L) - Linear component of factor A

**A(Q) - Quadratic component of factor A

The computation is shown in the last sample run in Appendix B where the above values are entered column by column as follows:

Column No.	Effect
1	A(L)
2	A(Q)
3	B(L)
4	B(Q)
5	A(L)B(L)
6	A(L)B(Q)
7	A(Q)B(L)
8	A(Q)B(Q)

The variance of error is estimated from the effects of A(Q), B(Q), A(L)B(L), A(L)B(Q), A(Q)B(L) and A(Q)B(Q) where the sum of squares of each effect is less than 0.9. Comparing with the results in 2.8.2, both calculations give the same conclusion, i.e., the effect of oxalic acid is significant and only linear component exists in the regression equation.

APPENDIX B

SAMPLE RUNS OF FADESN

----------FACTORIAL DESIGN----------

Y = B0 + B1*X1 + ... + BN*X1*...*XN

WHERE Y IS THE RESPONSE OR TEST RESULT
B0,...,BN ARE THE REGRESSION COEFFICIENTS
X1,...,XN ARE THE MAIN FACTORS
X1*...*XN IS THEIR (N-1)TH DEGREE INTERACTION

FOR EXAMPLE, A FACTORIAL DESIGN WITH 4 OBSERVATIONS;
2 FACTORS X1 AND X2; INTERACTION X1*X2; ALL FACTORS
AT 2 LEVELS -1(FOR LOWER LEVEL) AND +1(FOR HIGHER LEVEL),
THE DESIGN MATRIX X (WITH DIMENSION 4X3)
IS AVAILABLE AS DESIGN NO.1 IN THIS PROGRAM:

TEST NO.	X1	X2	X1*X2
1	-1	-1	1
2	1	-1	-1
3	-1	1	-1
4	1	1	1

Y = B0 + B1*X1 + B2*X2 + B3*X1*X2

HERE IS A LIST OF EXPERIMENTAL DESIGNS AVAILABLE IN
THIS PROGRAM (FOR DETAILS REFER TO CANMET REPORT MSL/MRP 84-150(IR))

DESIGN NO.	NO. OF LEVELS	DESIGN MATRIX DIMENSIONS (ROW X COLUMN)	POSSIBLE NO. OF FACTOR
1	2	4 X 3	2 OR 3
2	2	8 X 7	3,4,5,6 OR 7
3	2	16 X 15	4,5,6,7 OR 8
4	3	9 X 4	2,3 OR 4
5	4	16 X 5	2,3,4 OR 5

WHICH DESIGN YOU WANT (1,2,...OR 5,0 IF NONE OF THESE)?0
YOU ARE TO ENTER YOUR OWN EXPERIMENTAL DESIGN
ENTER THE NUMBERS OF LEVELS, ROWS AND COLUMNS: 2,8,7
THE LEVELS ARE TO BE REPRESENTED BY:
-1, 1 (2-LEVEL)
-1, 0, 1 (3-LEVEL)
-3, -1, 1, 3 (4-LEVEL)
MAXIMUM NO. OF LEVELS = 4

```
ENTER THE MATRIX ONE COLUMN AT A TIME:
    COLUMN( 1) =-1,1,-1,1,-1,1,-1,1
    COLUMN( 2) =-1,-1,1,1,-1,-1,1,1
    COLUMN( 3) =-1,-1,-1,-1,1,1,1,1
    COLUMN( 4) =1,-1,-1,1,1,-1,-1,1
    COLUMN( 5) =1,-1,1,-1,-1,1,-1,1
    COLUMN( 6) =1,1,-1,-1,-1,-1,1,1
    COLUMN( 7) =-1,1,1,-1,1,-1,-1,1

IS THIS THE DESIGN YOU WANT (Y/N) ?Y

INPUT  8 TEST RESULTS: 35.522,36.184,35.921,37.125,35.599,36.106
35.46,37.433

ANY REPEAT MEASUREMENTS (Y/N) ?Y
ENTER THE NUMBER OF NO. 1 REPEAT MEASUREMENT (ENTER 0 TO FINISH): 4
ENTER THE TEST RESULTS FROM THIS REPEAT: 36.334,36.759,36.616,36.227
ENTER THE NUMBER OF NO. 2 REPEAT MEASUREMENT (ENTER 0 TO FINISH): 0

THE VARIANCE OF EXPERIMENTAL ERROR =     .0605
THE NUMBER OF DEGREES OF FREEDOM   =        3

WHICH CONFIDENCE LIMIT 90 OR 95 ?95
```

REGRESSION COEFF.		SUM OF SQUARES	DEGREES OF FREEDOM	MEAN SQUARES	CRITICAL F(1, 3) 10.128 @95. %C.L. F-RATIO	SIGNIFICANCE
B 0:	36.169	10465.428				
B 1:	.543	2.361	1	2.361	.3900E+02	YES
B 2:	.316	.799	1	.799	.1320E+02	YES
B 3:	-.019	.003	1	.003	.4897E-01	NO
B 4:	.251	.504	1	.504	.8326E+01	NO
B 5:	.077	.047	1	.047	.7785E+00	NO
B 6:	-.019	.003	1	.003	.4771E-01	NO
B 7:	.116	.107	1	.107	.1763E+01	NO

```
WANT TO EXIT THE PROGRAM (Y/N) ?N
WHICH DESIGN YOU WANT (1,2,...OR 5,0 IF NONE OF THESE)?4
WANT TO EXAMINE THE MATRIX (Y/N)?Y

     DESIGN NO.      DESIGN MATRIX
     ----------      -------------
         4.

                     -1  -1  -1  -1
                     -1   0   0   0
                     -1   1   1   1
                      0  -1   0   1
                      0   0   1  -1
                      0   1  -1   0
                      1  -1   1   0
                      1   0  -1   1
                      1   1   0  -1

IS THIS THE DESIGN YOU WANT (Y/N) ?Y

INPUT  9 TEST RESULTS: 41.3,42.8,43,43,43.5,44.9,44.7,46,45,45.9

ANY REPEAT MEASUREMENTS (Y/N) ?N

ENTER THE VARIANCE OF EXPERIMENTAL ERROR AND ITS NUMBER
OF DEGREES OF FREEDOM (1 IF NONE OF THEM ARE KNOWN): 1


COLUMN NO.      SUM OF SQUARES      DEGREES OF FREEDOM
----------      --------------      ------------------
     1              16.007                  2
     2               1.307                  2
     3               1.402                  2
     4                .060                  2

FROM THE ABOVE TABLE, ENTER THE COLUMN NUMBER THAT CAN BE
USED TO ESTIMATE THE EXPERIMENTAL ERROR
(ONE AT A TIME): 3
NEXT (0 TO STOP)?4
NEXT (0 TO STOP)?0

THE MEAN VARIANCE OF EXPERIMENTAL ERROR =     .6106
THE NUMBER OF DEGREES OF FREEDOM        =        4

WHICH CONFIDENCE LIMIT 90 OR 95 ?95
```

REGRESSION COEFF.		SUM OF SQUARES	DEGREES OF FREEDOM	MEAN SQUARES	F-RATIO	CRITICAL F(2, 4) 6.944 @95. %C.L. SIGNIFICANCE
B 0:	44.122	17520.934				
B 1:	1.633	16.007	2	8.003	.1311E+02	YES
B 2:	.467	1.307	2	.653	.1070E+01	NO
ERROR		2.442	4	.611		

WANT TO EXIT THE PROGRAM (Y/N) ?N
WHICH DESIGN YOU WANT (1,2,...OR 5,0 IF NONE OF THESE)?5
WANT TO EXAMINE THE MATRIX (Y/N)?Y

DESIGN NO.	DESIGN MATRIX				
5.					
	-3	-3	-3	-3	-3
	-3	-1	-1	-1	-1
	-3	1	1	1	1
	-3	3	3	3	3
	-1	-3	-1	1	3
	-1	-1	-3	3	1
	-1	1	3	-3	-1
	-1	3	1	-1	-3
	1	-3	1	3	-1
	1	-1	3	1	-3
	1	1	-3	-1	3
	1	3	-1	-3	1
	3	-3	3	-1	1
	3	-1	1	-3	3
	3	1	-1	3	-3
	3	3	-3	1	-1

IS THIS THE DESIGN YOU WANT (Y/N) ?Y

```
INPUT 16 TEST RESULTS: 56,52,68,9,75,65,38,50,61,77,55,63,73,49,82,66

ANY REPEAT MEASUREMENTS (Y/N) ?N

ENTER THE VARIANCE OF EXPERIMENTAL ERROR AND ITS NUMBER
OF DEGREES OF FREEDOM (1 IF NONE OF THEM ARE KNOWN): 1
```

COLUMN NO.	SUM OF SQUARES	DEGREES OF FREEDOM
1	1001.113	3
2	667.012	3
3	400.513	3
4	99.012	3
5	400.513	3

```
FROM THE ABOVE TABLE, ENTER THE COLUMN NUMBER THAT CAN BE
USED TO ESTIMATE THE EXPERIMENTAL ERROR
(ONE AT A TIME): 4
NEXT (0 TO STOP) ?0

THE MEAN VARIANCE OF EXPERIMENTAL ERROR = 738.7625
THE NUMBER OF DEGREES OF FREEDOM        =       3

WHICH CONFIDENCE LIMIT 90 OR 95 ?90
```

REGRESSION COEFF.		SUM OF SQUARES	DEGREES OF FREEDOM	MEAN SQUARES	CRITICAL F(3, 3) 5.390 @90. %C.L. F-RATIO	SIGNIFICANCE
B 0:	58.688	55107.563				
B 1:	3.538	1001.113	3	333.704	.4517E+00	NO
B 2:	-2.888	667.012	3	222.338	.3010E+00	NO
B 3:	-2.238	400.513	3	133.504	.1807E+00	NO
B 5:	-2.238	400.513	3	133.504	.1807E+00	NO
ERROR		2216.288	3	738.763		

```
WANT TO EXIT THE PROGRAM (Y/N) ?N


WHICH DESIGN YOU WANT (1,2,...OR 5,0 IF NONE OF THESE)?0
YOU ARE TO ENTER YOUR OWN EXPERIMENTAL DESIGN
ENTER THE NUMBERS OF LEVELS, ROWS AND COLUMNS: 3,9,8
THE LEVELS ARE TO BE REPRESENTED BY:
         -1,  1            (2-LEVEL)
         -1,  0, 1         (3-LEVEL)
         -3, -1, 1, 3      (4-LEVEL)
MAXIMUM NO. OF LEVELS = 4

ENTER THE MATRIX ONE COLUMN AT A TIME:
    COLUMN( 1) =-1,-1,-1,0,0,0,1,1,1
    COLUMN( 2) =1,1,1,-2,-2,-2,1,1,1
    COLUMN( 3) =-1,0,1,-1,0,1, -1,0,1
    COLUMN( 4) =1,-2,1,1,-2,1,1,-2,1
    COLUMN( 5) =1,0,-1,0,0,0,-1,0,1
    COLUMN( 6) =-1,0,1,12,0,-2,-1,0,1
    COLUMN( 7) =-1,2,-1,0,0,0,1,-2,1
    COLUMN( 8) =1,-2,1,-2,4,-2,1,-2,1
ENTER THE NUMBER OF DEGREES OF FREEDOM FOR EACH COLUMN:
    COLUMN( 1) =1
    COLUMN( 2) =1
    COLUMN( 3) =1
    COLUMN( 4) =1
    COLUMN( 5) =1
    COLUMN( 6) =1
    COLUMN( 7) =1
    COLUMN( 8) =1

IS THIS THE DESIGN YOU WANT (Y/N) ?Y

INPUT  9 TEST RESULTS: 41.3,42.8,43,43.5,44.9,44.7,46,45,45.9

ANY REPEAT MEASUREMENTS (Y/N) ?N

ENTER THE VARIANCE OF EXPERIMENTAL ERROR AND ITS NUMBER
OF DEGREES OF FREEDOM (1 IF NONE OF THEM ARE KNOWN): 1
```

COLUMN NO.	SUM OF SQUARES	DEGREES OF FREEDOM
1	16.007	1
2	.269	1
3	1.307	1
4	.056	1
5	.810	1
6	.053	1
7	.853	1
8	.401	1

FROM THE ABOVE TABLE, ENTER THE COLUMN NUMBER THAT CAN BE
USED TO ESTIMATE THE EXPERIMENTAL ERROR
(ONE AT A TIME): 2
NEXT (0 TO STOP)?4
NEXT (0 TO STOP)?5
NEXT (0 TO STOP)?6
NEXT (0 TO STOP)?7
NEXT (0 TO STOP)?8
NEXT (0 TO STOP)?0

THE MEAN VARIANCE OF EXPERIMENTAL ERROR = .4070
THE NUMBER OF DEGREES OF FREEDOM = 6

WHICH CONFIDENCE LIMIT 90 OR 95 ?95

REGRESSION COEFF.		SUM OF SQUARES	DEGREES OF FREEDOM	MEAN SQUARES	CRITICAL F(1, 6) 5.987 @95. %C.L. F-RATIO	SIGNIFICANCE
B 0:	44.122	17520.934				
B 1:	1.633	16.007	1	16.007	.3932E+02	YES
B 3:	.467	1.307	1	1.307	.3210E+01	NO
ERROR		2.442	6	.407		